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# EVALUATION OF STABILITY OF SYSTEM WITH NEURAL CONTROLLER

*Yu. Hirnyak, O. Ivakhiv, M. Nakonechnyi, T. Repetylo*

L'viv Polytechnic National University, 12 Bandera Street, L'viv, Ukraine, 79013,

## ABSTRACT

The controllers which are built using classical methods in the process of system operation do not provide full adequacy with controlled values of nonlinear objects. We consider the features of dynamic neural network and the reference mathematical models construction which carried the review of neurons activation functions and feasibility of using gradient algorithms including the Levenberg-Marquardt algorithm for training of dynamic neural networks. Obviously the control action of such dynamic system is corresponded with the difference between output signals of the non-linear object and the chosen reference. On contrary to a typical structure it was proposed to put these signals on two separate inputs. The comparison of obtained errors in traditional and proposed structures of input circuit of neural controller showed that the last one is the most effective in quality and productivity sense. Also the stability of the system supported by neural controller with two separated inputs is discussed.

***Index Terms -*** Neural, network, non-linear, object, system, control, controller, algorithm, training.

## 1. INTRODUCTION

As it is known, the objects of the automatic control systems are mostly described by nonlinear differential equations, and only at the insignificant deviations from basic values such system can be approximately considered as linear [1-3]. In addition, often nonlinear phenomena are not taken into account in the mechanical systems, such as a dry friction, influence of backlashes and limitations. The mathematical means of the nonlinear systems analysis is corresponded with the investigation of nonlinear differential equations. The theory of solving them is based on the usage of the specialized numeral approaches; each of them is worked out for some definite type of equation which describes the concrete system. It's complicated to decide on nonlinear differential equation entailed elaboration of such methods, which allow forming conclusions relatively to the nature of processes which take a place in the system. Thus nonlinear descriptions of the real elements at the system replaced by idealized. It depends both on description of nonlinear element and on the chosen method of analysis of the system. Nowadays it appears that the linear dynamic objects are quite thoroughly researched and the methods of their control have been worked out [1-3]. As it concerns nonlinear objects, for solving of the nonlinear differential equations, beside of the analytical and graphical solving methods, widely used simulation methods, in particular using the methodology of theory of artificial neural networks [4-8]. That allows taking into account afore-mentioned non-linearity, realizing certain functional descriptions by means of superposition from linear combination of activation functions. In particular, robot arm leaded by DC electromotor with control of the rotor circuit is one of such nonlinear object examples. Position of the robot arm is fully determined by the turn angle of the motor shaft witch is connected to the gear. In this case it is necessary to add to the moment of inertia of the rotor the applied moment of inertia

of the robot arm and to the moment of operating forces – the moment of resistance which acts on the robot arm, is applied to the motor shaft. So, for creation of dynamics equations of controlled object it is needed to combine parts which take into account the applied mass of arm and acted forces applied to it. Also, motor is loaded with the moment of viscid friction and moment of the robot arm rotation which depends of the arm position. Consequently, resulting equation of DC motor with independent activation takes into account the moment of viscid friction and moment of the robot arm rotation is nonlinear differential equation of the second order [9]. Linearizing of the obtained equation results in its considerable simplification, however here taken into account non-linearity of the motor descriptions as one of features that more essential of all influences on the motor behavior, both in transient and in stationary types of operation. Therefore in the automatic control system of the robot arm as a nonlinear object the neural controller is incorporated.
To realize a settled target it is necessary to follow some important steps:

1. The first of all a real nonlinear object or its created mathematical model is necessary. It has to form the corresponding structure of controller in accordance with the chosen control law.
2. To choose the mathematical model of reference, coming from the desirable dynamics of controlled object which sets a duration of transient process, possible overloading etc. as indexes of adjusting law. Such a reference provides creation of the proper sequence which is used for training of neural controller. It is necessary to have numerical sequences on an input and output of object. All of numerical sequences are written down in memory of machine. The purpose of training consists in picking up of obtained weighing coefficients for all of neuron realizations in the neural controller, which ensures minimal differences between the output signals of an object and a reference.
3. After completion of training procedure the desired structure is prepared, the values of weighing coefficients, number and duration of delays, are carried in the environment of Simulink. Coincidence of output signals of a reference and an object have to be compared for a system which is controlled by the synthesized controller.
4. And at last the controller is combined with an object and duration of transient process, duration of delays, quality of reacting on disturbance (working off) as well the stability of control are expected.

To achieve this aim both the reducing of discrepancy between the output signals of a system and a reference by improving the architecture of the neural controller and stability are investigated.

## 2. CONTROLLER STRUCTURE DESIGNING

Following a differential equation of system dynamics, it is possible simply to define such configuration of input-outputs of dynamic neural controller which provides the reproducing of process which coincide the solving of this equation. Such dynamic neural network structure for the reproducing of the linear differential equation is determined by application to this equation presented in difference form of Z-transformation, and nonlinear dependences which are included in the structure of nonlinear differential equation will be realized by introduction of elements with the nonlinear functions of activity between inputs and outputs of the hidden layers of dynamic neuro network [8].
Using the results [9] of experimental research of the dynamics of the robot arm that moves in one of the six possible directions it was established that its dynamics can be described by a nonlinear equation of the second order, namely, having a free member of sinusoidal character:

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+10\sin y=u(t), \tag{1}$$

where $y$ – is the reaction of controlled object to the input control signal $u$, i.e., in the static state, the dependence between the input and the output variables is nonlinear:

$$y=\frac{1}{10}\arcsin u. \tag{2}$$

It is suggested that its operation should be based on the use of neural networks ideology as a structure which corresponds to a proportional-integral-differential control law regulator [2, 3], because the control signal contains the change in the current value of the controlled variable of the object, its prediction (i.e., speed of change) and the previous state, i.e., described by the equation:

$$u(t)=K_q\frac{d\Delta y(t)}{dt}+K_n\Delta y(t)+K_i\int_0^{T_i}\Delta y(t)dt, \tag{3}$$

where $u(t)$ is the output signal of a controller; $\Delta y(t)$ is the difference signal at the input of a controller; $T_i$ is the integration period (its value chosen on the user's demand); $K_q$, $K_n$ and $K_i$ are coefficients at differential, current and integral components of the formed control signal, respectively.

Transition from the linear equation (3) to the structure of the neural network, which reproduces it, ensures the applicability of the functional analogy between the structure of linear neuron and the structure of digital filter. This makes it possible to use the mathematical apparatus of the digital filter theory for creating networks. So, the proposed technique is based on the discrete presentation of this law and determination of the required number of the delay lines. In operator form, this equation will look as follows:

$$U(s)=\left(K_q s+K_n+\frac{K_i}{s}\right)Y(s). \tag{4}$$

That's why the controller transfer function can be described as follows:

$$W(s)=U(s)/Y(s)=K_q s+K_n+K_i/s. \tag{5}$$

Dynamic network operates as a discrete system, so this law should be applied in a discrete form using the replacement of the operator $s$ by the operator $z$, i.e. $s=\left(1-z^{-1}\right)/(\Delta t)$, namely:

$$W(z)=K_q(1-z^{-1})/\Delta t+K_n+(K_i\Delta t)/(1-z^{-1})=$$

$$=\frac{1}{(1-z^{-1})}\left[\frac{K_q}{\Delta t}(1-z^{-1})^2+K_n(1-z^{-1})+K_i\Delta t\right]=$$

$$=\frac{1}{(1-z^{-1})}\left[\left(\frac{K_q}{\Delta t}+K_n+K_i\Delta t\right)-\left(2\frac{K_q}{\Delta t}+K_n\right)z^{-1}+\frac{K_q}{\Delta t}z^{-2}\right]=$$

$$=\frac{w_{11}+w_{12}z^{-1}+w_{13}z^{-2}}{1-w_{14}z^{-1}}, \tag{6}$$

here, $w_{11}=\left(K_q+K_n\Delta t+K_i\Delta t^2\right)/\Delta t$; $w_{12}=-\left(2K_q+K_n\Delta t\right)/\Delta t$; $w_{13}=K_q/\Delta t$; $w_{14}=1$ – are the weighing coefficients of the designed neural network.

Weighting coefficients of the network related to the coefficients of the controller equation are estimated by the ratio: $K_q=w_{13}\Delta t$; $K_n=-\left(w_{12}+2w_{13}\right)$; $K_i=\left(w_{11}+w_{12}+w_{13}\right)/\Delta t$, where $\Delta t$ is the sampling step. The differential equation which corresponds to the expression (6), at the $k$-th step of controlling will look as follows:

$$u_k=w_{11}\Delta y_k+w_{12}\Delta y_{k-1}+w_{13}\Delta y_{k-2}+w_{14}u_{k-1}. \tag{7}$$

According the equation (7) it was got [8] a single-layer network structure (Fig. 1) of the four-input neural network with one adder, one linear circuit of activation, four multipliers and three delay lines.

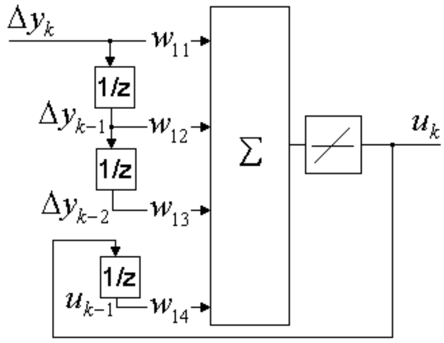



Fig. 1. Structure of the neural controller with one-port input circuit

Here, the signal at its output is $u = F(S) = kS$, where $F$ is the activation function of the network (in this case $F(S)$ is linear); $S = \sum_i w_{1i} x_i$ is the signal at the input of the activation circuit; $x_i$ and $w_{1i}$ are the *i*-th signal at the input of the first layer adder of the neural network and its weight coefficient, respectively (in this case $x_1 = \Delta y_k$, $x_2 = \Delta y_{k-1}$, $x_3 = \Delta y_{k-2}$, $x_4 = u_{k-1}$). Input signals of the adder are formed by four multipliers having weights $w_{11}$, $w_{12}$, $w_{13}$ and $w_{14}$, respectively, and by three delay lines.

## 3. TRAINING OF THE NEURAL CONTROLLER

Control system is based on usage of artificial neural network technologies [4] as the universal approximating device, which after the training process based on defined sequences, can accommodate the input data in order to get at the output of the network the values as close as possible to the related output signals given in the form of the objective function. At the stage of simulation research the neural network consists of two subnets: the model of object and the model of controller. While using the back propagation algorithm, the error passes through a subnet of the object without changes, and in the subnet of controller it provides the correction of weighing coefficients. The desired character of motion is set by the model of a reference. Therefore, the weighing coefficients of the chosen neural controller architecture concern the process of training. The degree of the created system approaching the reference is estimated by establishing the disagreement between the outputs signals of this system and the reference at the same input signal, for example, minimizing the middle square error estimation. Controller complements an object would be maximally equal to a reference (ideally $y_{rk} = y_k$). In the known neural networks the deviation between the object's output signal $y_k$ and input signal $r_k$ is placed at the controller input.

For training of the controller with the selected architecture (Fig.1) the reference which can be described by the linear differential equation was used to determine the weight coefficients of the network. In order to ensure the equality between the input and the output signals in a static state, their coefficients are set to be the same. Namely:

$$\frac{d^2 y_r}{dt^2} + 6\frac{dy_r}{dt} + 9y_r = 9r_k, \tag{8}$$

where $r_k$ and $y_r$ are the input and the output signals of the reference.
Note that the reference matching proceeds in compliance with certain conditions:

- order of differential equation should coincide with the order of the dynamic equation of the controlled object (in this case, of the second order); constants are chosen from the condition of convergence between the output signal of the reference and the output signal of the object in the static state;
- constants at the first derivative and the free component of the equation are chosen based on the desired form of the transition process.

In the generalized structure of the automatic control system (Fig. 2) the neural controller supplements the non-linear object in such a way that the formed system should maximally correspond to the reference (ideally $y_k = y_{rk}$) in case of applying any acceptable sequence $r_k$.

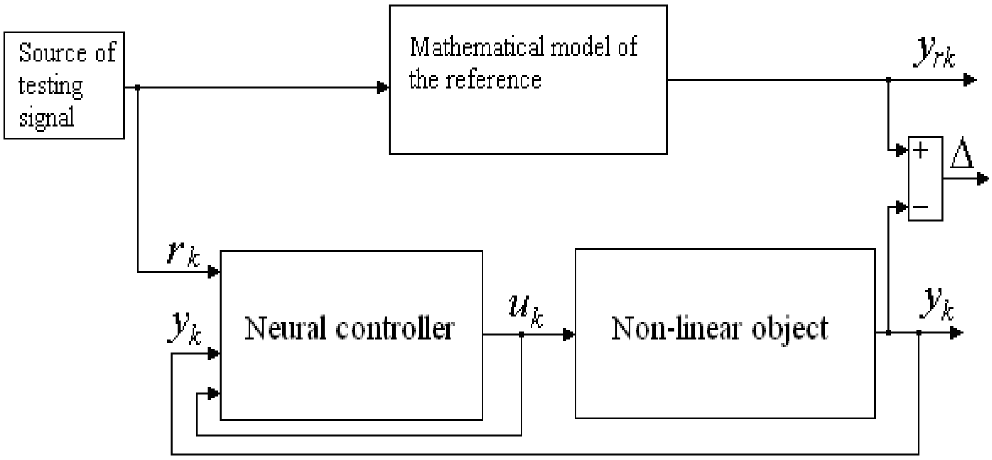



Fig. 2. An automatic control system with a neural controller

Since it is necessary to know the input and output signals of a neural network in order to train it, the neural controller can be trained if we simultaneously know the reference signal at the input of neural controller (sequence $r_k$); the feedback signal from the output of the object (sequence $y_k$); the output signal of the neural controller (sequence $u_k$, that is applied to the input of an object). All three mentioned above sequences are simultaneously unknown prior to training the neural network. If you set the input signal (sequence $r_k$), then assuming that $y_k = y_{rk}$, based on the mathematical model of the reference you can determine the sequence $y_k$, but then the sequence $u_k$ remains unknown. If we consider that the sequence $u_k$ is known and then apply it to the input of the object, then at its output we get the sequence $y_k$, but the sequence $r_k$ remains unknown.
Simulations within the Simulink of MATLAB package [9] were carried out in accordance with experimental data of the Nokia Puma robot. In the nowadays known papers (in particular, [4-7]), the difference between the output signal of a system and the reference signal is applied directly to the input of a neural controller. The comparison of the output signals from the reference and the object of the system controlled by a neural controller trained by the ratio (7), points to a low efficiency of the system. That is, the controller which used signal differences $\Delta y_k$ for training the neural network does not provide the required dynamic parameters (Fig. 3,a).

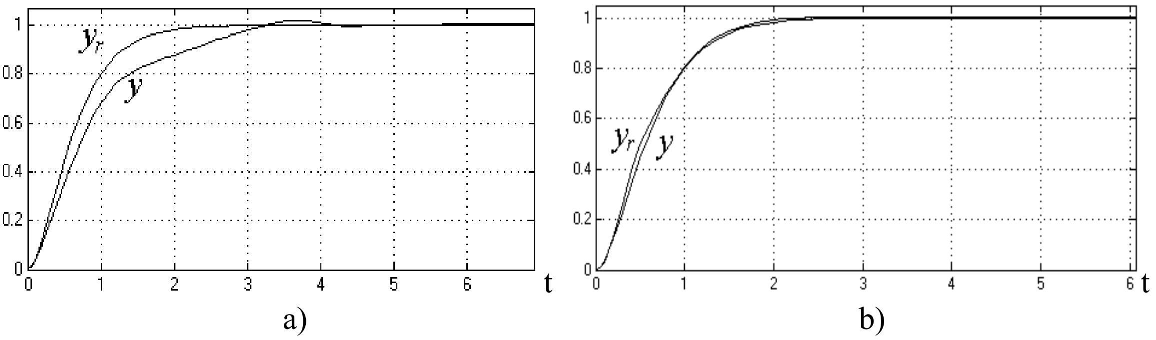



Fig.3. Comparison of the $y_r$ and *y* output signals of the reference source and the system controlled by the neural controller (*a* – with one-port and *b* - with two-port input, respectively)

In order to improve the dynamic characteristics on contrary to the such traditional one-port way of feedback realization it was planned to test the efficiency of the control system operation when using the two-port input circuit of the controller, namely, instead of the difference between the output signals of the system and the testing source to establish these signals separately through the appropriate delay lines in the controller [10]. So, in this case an output signal of a system is applied to one input, while the reference signal – to the other input. Simulation research has shown that in this case the discrepancy between the output signals of the reference and the system is smaller than in case of using a traditional architecture of input circuit of the neural controller. Here (Fig. 4) the one-port input circuit of the regulation error $\Delta y_k$ and its previous (delayed) values $\Delta y_{k-1}$, $\Delta y_{k-2}$ were replaced with the two-port input circuit of the controller with their own weights of separated signals, i.e., the values of $r_k$, $r_{k-1}$, $r_{k-2}$ separately from the values of $y_k$, $y_{k-1}$, $y_{k-2}$.

The equation that describes the operation of the neural controller with separated inputs looks as follows:

$$u_k = w_{11}r_k + w_{12}r_{k-1} + w_{13}r_{k-2} + w_{14}y_k + w_{15}y_{k-1} + w_{16}y_{k-2} + w_{17}u_{k-1}. \tag{9}$$

Obviously, at $w_{14} = -w_{11}$, $w_{15} = -w_{12}$, $w_{16} = -w_{13}$ the above scheme is equivalent to the previous one (Fig. 1), but in the process of training, the coefficients for individual inputs are set independently of each other, which, if needed, makes it possible to supply the control action to the object outside the circle of feedback. The value of the weight coefficient $w_{17}$ is assumed to be always fixed and equal to 1.

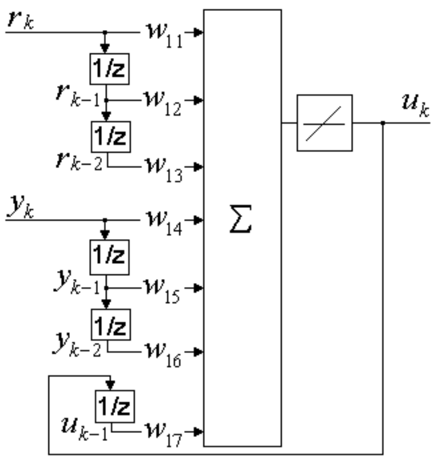



Fig. 4. Structure of a neural controller with two-port input circuit

Comparison of the differences between the output signals of the reference and the object controlled by such controller (Fig. 3,b) shows that in this case the system possesses the better dynamic characteristics than the previous one. It confirms the feasibility of using the controller with two-port input circuit. Hence, in the worst case, the difference between output signals was 14.3% for the usual structure of controller and 7.1% for the controller with separate inputs. The duration of training was the same for both structures and equal no more than 1 minute.

## 4. SYSTEM STABILITY EXAMINATION

For evaluation of the system stability using a classical methods [2] the first of all it is necessary to linearize the non-linear equiation of the controlled object (1) at the most unstable point of object, i.e. at the greatest slope of its static function. So, it is as follows:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 10y = u\ . \tag{10}$$

Its transfer function is as follow:

$$W_o(s) = \frac{1}{s^2 + 2s + 10}\ . \tag{11}$$

At the second step we transform the controller configuration into the closed circuit that allows applying the methodology of linear systems analizing. Accordingly to the equation (7) was added the new one

$$0 = w_{14}r_k + w_{15}r_{k-1} + w_{16}r_{k-2} - w_{14}r_k - w_{15}r_{k-1} - w_{16}r_{k-2}\ . \tag{12}$$

Then we received as follow:

$$\begin{aligned} u_k &= (w_{11} + w_{14})r_k + (w_{12} + w_{15})r_{k-1} + (w_{13} + w_{16})r_{k-2} - \\ &- w_{14}(r_k - y_k) - w_{15}(r_{k-1} - y_{k-1}) - w_{16}(r_{k-2} - y_{k-2}) + w_{17}u_{k-1}. \end{aligned} \tag{13}$$

Taking into account that $\Delta_{yk} = r_k - y_k$, we obtain a following expression

$$\begin{aligned} u_k &= (w_{11} + w_{14})r_k + (w_{12} + w_{15})r_{k-1} + (w_{13} + w_{16})r_{k-2} - \\ &- w_{14}\Delta y_k - w_{15}\Delta y_{k-1} - w_{16}\Delta y_{k-2} + w_{17}u_{k-1}, \end{aligned} \tag{14}$$

or other way $u_k = u_{1,k} + u_{2,k}$, here

$$u_{2,k} = -w_{14}\Delta y_k - w_{15}\Delta y_{k-1} - w_{16}\Delta y_{k-2} + w_{17}u_{2,k-1}\ , \tag{15}$$

$$u_{1,k} = (w_{11} + w_{14})r_k + (w_{12} + w_{15})r_{k-1} + (w_{13} + w_{16})r_{k-2} + w_{17}u_{1,k-1}\ . \tag{16}$$

In this case neural controller is being as two-sectional one with two identical parts (Fig.5).
The output signal of the first subnet depends only on the control signal and the second one – on the difference between the control signal and the output signal of the object. Functionally this structure is identical with the previous one (Fig.4) that allows using it for the behaviour investigation of the structure with separate inputs of two-port input circuit.
Transfer functions of these subnetworks are as follows:

$$W_1(z) = \frac{U_1(z)}{r(z)} = \frac{(w_{11} + w_{14}) + (w_{12} + w_{15})z^{-1} + (w_{13} + w_{16})z^{-2}}{1 - w_{17}z^{-1}} \tag{17}$$

$$W_2(z) = \frac{U_2(z)}{r(z)} = \frac{-w_{14} - w_{15}z^{-1} - w_{16}z^{-2}}{1 - w_{17}z^{-1}}\ , \tag{18}$$

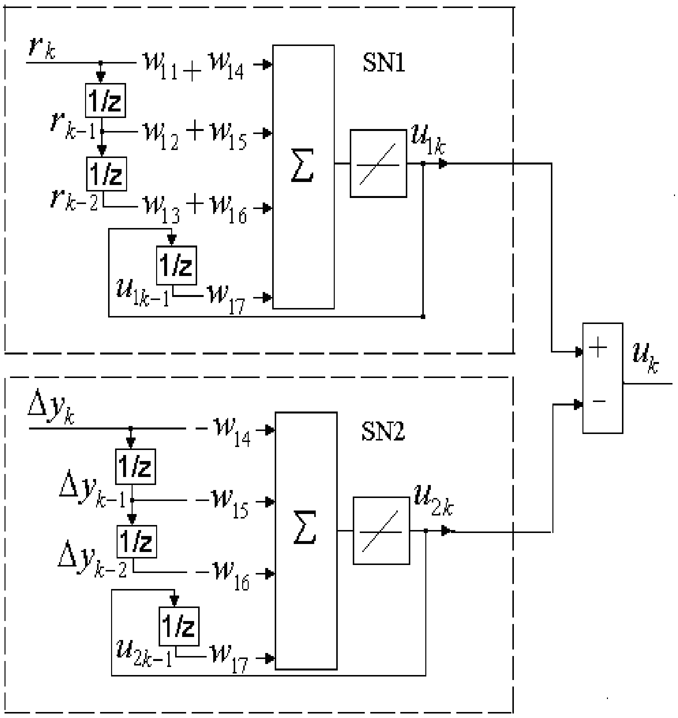



Fig. 5. Neural controller structure chart with two subnets

Let us analyze the automatic control system described by the modified chart (Fig.6) using the continuous representation. Therefore taking into account the correspondence between the operator *s* and *z* for continuous and discrete presentations accordingly, i.e. $z = 1/[1 - s(\Delta t)]$, transform expressions (17) and (18). Then we obtain following:

$$W(s) = \frac{Y(s)}{R(s)} = \frac{W_0(s)(W_1(s)+W_2(s))}{1+W_0(s)W_2(s)} = \frac{(K_{d1}+K_{d2})s^2+(K_{p1}+K_{p2})s+K_{i1}+K_{i2}}{s^3+(K_{d2}+2)s^2+(K_{p2}+10)s+K_{i2}}. \quad (19)$$

One part with transfer function $W_1(s)$ allows to form the first component $U_1$ of the control signal and another one (as the closed feedback serial connection chains with transfer functions $W_2(s)$ and $W_0(s)$, respectively) – the second component $U_2$.

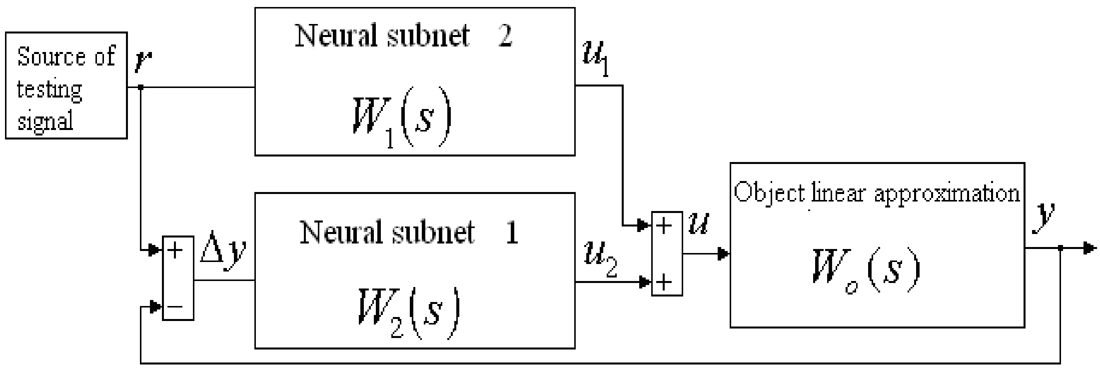
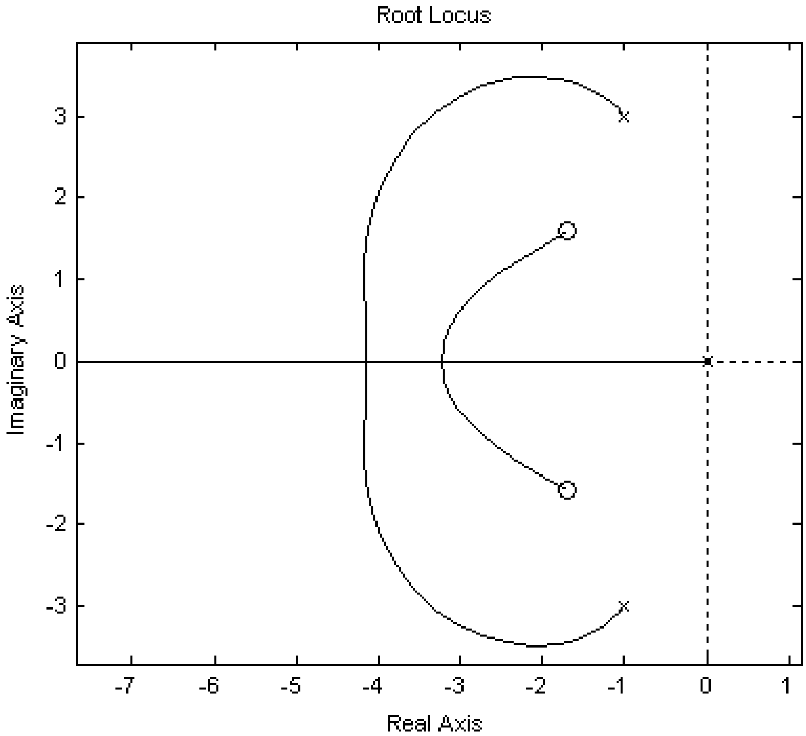


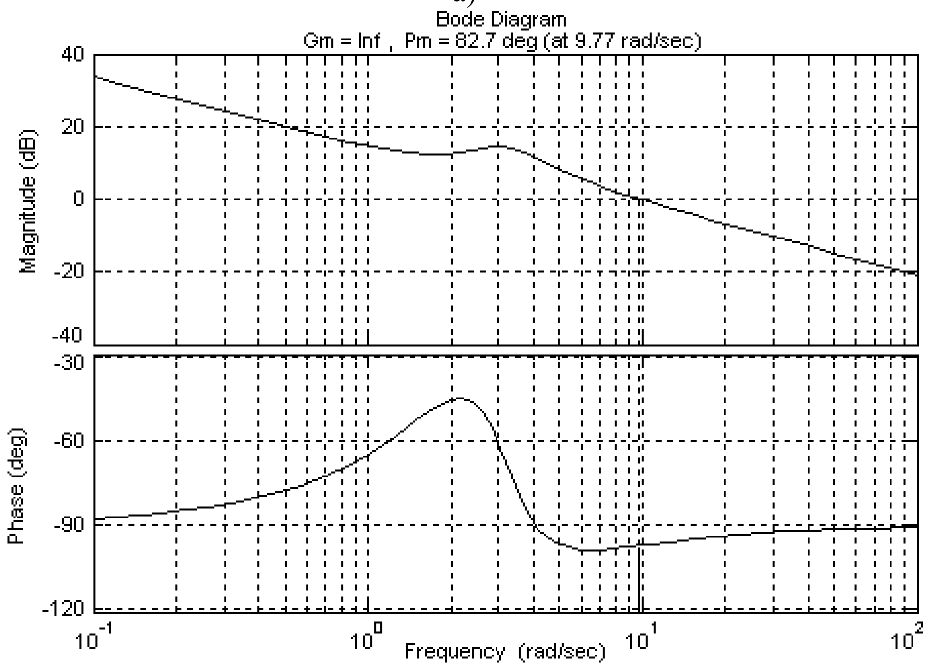

Fig. 6. The automatic control system with a neural controller as two subnet presentation

Such presentation allows to consider created system as continuous one and therefore to analyze system with Nyquist stability criterion using [2]. The trajectories of its characteristic equation roots lie in the left half-plane of rectangular coordinate system (Fig.7,a) and it means that the well-trained system is considered as stable at any values of its transfer coefficient.

The frequency characteristics (Fig.7,b) of the open-loop system also confirm stability of the system (a phase static margin is equal to 82˚).




a)




b)

Fig. 7. The root hodograph (a) and frequency characteristics (b) of an automatic control system with neural controller reduced to continuous version

These both conclusions are true for linear and non-linear controller after linearization of trained neural controller which allows using linear methodology for non-linear system dynamics investigation.

## 5. CONCLUSIONS

1. It is showed that the dynamic neuro network structure which reproduces the linear differential equation is similar to the structure of digital filter and determined by Z-transform application to this equation.
2. It is determined that if to follow the differential equation by which the controlled object is described, it is possible simply to define such configuration of inputs for dynamic neural network, which reproduces the processes that give the solvation of this equation.
3. It is determined that nonlinear dependences which are included in the structure of nonlinear differential equation can be realized by elements with the nonlinear activation functions implemented between inputs and outputs of the hidden layers within dynamic neuro network.
4. Analysis of the simulation results shows that the system using the controller with two-port input circuit possesses better dynamic characteristics than the system with regular structure of controller input circuit, i.e., its response is closer to one defined by the reference.
5. Stability of automatic control system with integrated neural controller confirmed by simulation results.
6. Among the training algorithms of artificial neural networks the most productive is the Levenberg-Markvardt algorithm.

## REFERENCES


[1] E.G. Woschni, Messdynamik, Leipzig: Hirzel Verlag, 1964

[2] Graham C. Goodwin, Stefan F. Graebe, Mario E. Salgano, Control System Design, Upper Saddle River, New Jersey: Prentice Hall, 2001

[3] K. S. Fu, R.C. Gonzalez, C.S.G. Lee, Robotics: Control, Sensing, Vision, and Intelligence. McGraw-Hill Education (India) Pvt Limited, 1988

[4] M. Norgaared, O. Ravn, N. Poulsen, L. Hansen, Neural Networks for Modelling and Control of Dynamic Systems, London: Springer, 2000

[5] H. Al-Duwaish, S.Z. Rizvi “Design of a Neuro-Controller for Multivariable Nonlinear Time-Varying Systems,” WSEAS Transactions on Systems and Control, Issue 9, Volume 5, September 2010, P.711-720

[6] S.M. Sharun, M.Y. Mashor, M.N. Norhayati, W.N. Hadani, S. Yaacob “Adaptive Neuro-Controller for Three-Axes Attitude Control of Innovative Satellite,” International Journal of Research and Reviews in Artificial Intelligence, Vol. 1, No. 1, March 2011, P.1-6

[7] S.M. Sharun, M.Y. Mashor, W.N.H. Wan Jaafar, N. Mohd Nazid, S. Yaacob “Adaptive Neuro-Controller Based on Hybrid Multi Layered Perceptron Network for Dynamic Systems,” International Journal of Control Science and Engineering, 2012, 2(3), P.34-41

[8] O. Bazylevych, O. Ivakhiv, Yu. Hirnyak, M. Nakonechnyi, “Neural Controller for Designing Nanopositioning Systems,” Proceedings of the International Conference on Precision Measurement, Ilmenau: Ilmenau University of Technology, 08-12 September, 2008, P.41-42.

[9] Network Toolbox Documentation [Online] The MathWorks, inc. Available: http://www.mathworks.com/help/nnet/index.html

[10] Yu. Hirnyak, O. Ivakhiv, M. Nakonechnyi, T. Repetylo, “Control System of Robot Movement,” Proceedings of the 7 IEEE International Conference on Intelligent Data Acquisition and Advanced Computing Systems (IDAACS), Volume 1, Berlin, Germany, September 12 – 14, 2013, P.334 – 337.

## CONTACTS

Prof. Dr.-Ing. habil. O. Ivakhiv oresti@polynet.lviv.ua